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Traffic Flow Modelling Around Roundabouts Using Navier–Stokes and Advection–Diffusion Equations

Momanyi Mogire Krifix[1], Shichikha Maremwa[2], Kandie Joseph[3], Lucy Jerop Ngetich[4]

[1,2,3,4] Department of Mathematics and Computer Science
University of Eldoret, Kenya

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Abstract: **Urban roundabouts are safer than signalised junctions yet remain prone to congestion under unbalanced demand and operational incidents. Existing microscopic and macroscopic approaches rarely capture, within a single framework, the tightly coupled evolution of traffic velocity and density needed for robust design and control. We address this gap with a coupled Navier–Stokes and Advection–Diffusion (NS–AD) model on an annular domain, adopting a barotropic closure $p = a\rho$ and a conservative incident force $k_{\text{obs}}\mathbf{r}/\| \mathbf{r} \|$. Convection is discretised with a QUICK/TVD flux (MUSCL with a van–Leer limiter), while diffusion, sources and viscosity are advanced by a semi-implicit Crank–Nicolson step, enabling high-resolution fronts without spurious oscillations over long horizons. Simulations reproduce two robust signatures: a persistent annular congestion ridge in density coincident with the circulating ring, and a co-located speed amplification that exhibits an upstream/downstream braking/acceleration asymmetry governed by $-a\nabla \ln\rho$. Probe histories display a negative $(|v|, \rho)$ phase slope during the transient, and a quasi-1D azimuthal reduction explains the observed structures and yields a falsifiable ridge-thickness law $\delta \sim D/u$. Difference maps between incident and baseline scenarios isolate the causal footprint of the incident as outward mass migration to the ring and momentum gain along preferred circumferential paths. We conclude that a composite potential $a\ln\rho + \Phi$ succinctly explains outward push, pressure support and viscous/diffusive regulation, while the numerical pairing QUICK + TVD with Crank–Nicolson provides a stable, accurate basis for design studies. For policy and practice, the results support rapid incident clearance near the central island, targeted entry metering on feeder approaches, and low-cost geometric/control provisions, such as short bypasses and advisory speeds, that reduce the effective barrier $a\ln\rho$ and add diffusion pathways. Future work should calibrate $(a, D, \nu, S(\theta), k_{\text{obs}})$ against field data using ridge width, phase slope, relaxation time and entry/exit counts, extend to multi-lane and multi-class settings, assimilate real-time data for model-predictive control, and couple to emissions and safety surrogates to assess broader impacts.**

Keywords: **Advection–diffusion of density; Crank–Nicolson time integration; Incident (stalled-vehicle) modelling; Navier–Stokes traffic modelling; QUICK/TVD convective scheme; Roundabout congestion.**

I. INTRODUCTION

A. Background Information

Traffic congestion remains one of the most pressing challenges in modern urban transportation systems [1, 2]. Roundabouts, which transform complex junctions into sequential T-intersections [3], have been widely adopted as an effective means of improving traffic safety and reducing delays compared to signalized intersections [4, 5]. They are particularly important in developing countries where the cost of traffic light infrastructure and maintenance is prohibitive [6]. However, despite their advantages, roundabouts often experience operational inefficiencies under heavy or unbalanced traffic volumes, leading to severe congestion, increased travel times, and elevated emissions [7, 8].

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Globally, road congestion continues to escalate due to increasing motorization [9, 10]. For instance, in Malaysia, 2.59 million new vehicles were registered between 2016 and 2020, averaging over 519,000 per year, with congestion cited as a leading cause of travel delays and productivity losses [7]. In Kenya and other African countries, the rapid urban population growth and rising vehicle ownership exacerbate gridlock at unsignalized intersections, particularly during peak hours. Simulation studies confirm that roundabouts, while safer, can become bottlenecks when yielding rules and driver heterogeneity are not effectively modeled [6]. Furthermore, traditional gap-acceptance models are often limited in capturing the nonlinear and stochastic dynamics of roundabout traffic, especially at multi-lane facilities, where driver behavior variability plays a significant role in throughput and delay [5].

According to the World Bank, traffic congestion costs developing economies between 2%–5% of their GDP annually due to lost productivity [11, 12], fuel wastage [13], and environmental damage [14]. In Nairobi, for example, average commuters lose nearly an hour per day in traffic [15, 15], and unsignalized roundabouts are frequently identified as congestion hotspots [16]. These realities underscore the need for robust and physics-based models that can more accurately capture the dynamics of vehicle interactions in such critical infrastructures.

Recent advances in traffic flow theory suggest that continuum-based models derived from fluid dynamics, such as the Navier–Stokes and Advection–Diffusion equations, provide a powerful framework for understanding velocity distributions, density fluctuations, and propagation of disturbances in roundabout traffic flow [17, 7]. By treating traffic as a compressible fluid, these models overcome the limitations of classical gap-acceptance methods and cellular automata, offering scalability and analytical rigor. Moreover, parameter estimation and numerical fitting methods make it possible to calibrate such models against real-world data, ensuring their applicability to heterogeneous urban environments.

This proposed study seeks to develop a Navier–Stokes and Advection–Diffusion based model to analyze the specific influence of roundabouts on vehicle velocity. By integrating advanced mathematical formulations with empirical fitting, the study aims to provide new insights into congestion dynamics and inform sustainable urban traffic management. The outcomes are expected to assist policymakers and urban planners in designing safer and more efficient roundabouts, reducing delays, and enhancing mobility in rapidly growing cities.

B. Contribution

This study introduces a unified continuum formulation that advances roundabout analysis beyond the separate strengths of microscopic cellular automata and macroscopic scalar conservation laws by evolving velocity and density in lockstep under a single NS–AD model with $p = a\rho$. It supplies the first minimal, mechanistic representation of incidents as a conservative potential Φ that acts directly on momentum while remaining interpretable and tunable, thereby enabling clean causal contrasts between incident and baseline fields. It contributes a numerically robust yet sharp-resolving solver that couples QUICK/TVD convection to Crank–Nicolson diffusion/viscosity, preserving high order in smooth regions and monotonicity near steep fronts. It further offers an azimuthal reduction that clarifies the ring-level balances and yields the explicit scaling $\delta \sim D/u$ for ridge thickness, which, together with a set of calibration-ready diagnostics peak radius, FWHM width, ring-averaged speed, probe phase slope and relaxation time—turns the continuum formulation into a falsifiable, data-driven tool for design and control. In doing so, the work closes a methodological gap by providing a single, physics-based pipeline that explains dense platoons, capacity loss under imbalance and downstream recovery within the same mathematical picture and delivers actionable predictions for incident management and metering.

II. RELATED WORKS

A. Theoretical Formulation

This section outlines the theoretical foundation for modelling traffic flow around roundabouts by combining the Navier–Stokes and Advection–Diffusion (AD) equations. The analogy between traffic dynamics and fluid flow allows the adaptation of conservation principles of mass and momentum, enabling the analysis of vehicle velocity, density variations, and congestion dynamics.

1) Continuity Equation

The conservation of vehicles is represented by the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}$$

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where $\rho(x, y, t)$ denotes traffic density and $\mathbf{v}(x, y, t) = (u, v)$ is the velocity vector. This ensures vehicle density changes only through inflow or outflow at any point in the road network.

2) Navier–Stokes Analogy for Traffic Flow

The Navier–Stokes equation, adapted to traffic, models the conservation of momentum:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v} + \mathbf{F}, \tag{2}$$

where p is the pressure induced by vehicle interactions, ν is the traffic viscosity (resistance to flow), and $\mathbf{F}$ represents external forces such as obstacles or breakdowns. In roundabouts, the obstacle force can be expressed as:

$$\mathbf{F}_{\text{obs}} = k_{\text{obs}} \frac{\mathbf{r}}{\|\mathbf{r}\|}, \tag{3}$$

where $\mathbf{r}$ is the distance vector to the obstacle and k_{obs} is the repulsion coefficient.

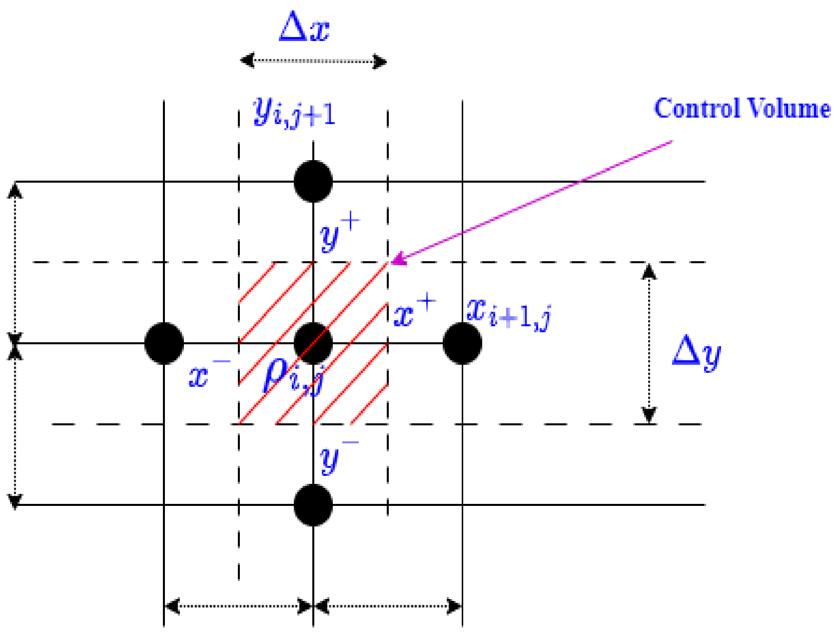

Fig. 1: Control volume illustrating velocity and density gradients within a roundabout segment.

3) Advection–Diffusion Dynamics

The AD equation accounts for the transport and dispersion of vehicle density:

$$\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho = D\nabla^2 \rho + S, \tag{4}$$

where D is the diffusion coefficient capturing fluctuations due to lane changes or driver variability, and S represents sources and sinks (e.g., vehicle entry/exit).

4) Coupled Navier–Stokes–Advection–Diffusion Model

To capture both density and velocity dynamics in roundabouts, equations (2) and (4) are combined:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho &= D\nabla^2 \rho + S, \\ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} &= -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v} + k_{\text{obs}} \frac{\mathbf{r}}{\|\mathbf{r}\|}. \end{aligned} \tag{5}$$

This coupled system captures (i) conservation of vehicles, (ii) propagation of velocity disturbances, (iii) diffusion of density waves, and (iv) effects of local disruptions like accidents or high inflow rates.

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5) Basic Assumptions

To simplify the analysis:

1. Steady-state flow is assumed for long-term averages.
2. Vehicles are homogeneous in size and behavior.
3. Two-dimensional flow (x, y) is considered, reflecting realistic roundabout geometry.
4. Road geometry outside the roundabout is constant.

6) Conceptual Framework

Fig. 2 presents the conceptual model, integrating Navier–Stokes and AD formulations to describe vehicle flow, density propagation, and congestion buildup within the roundabout.

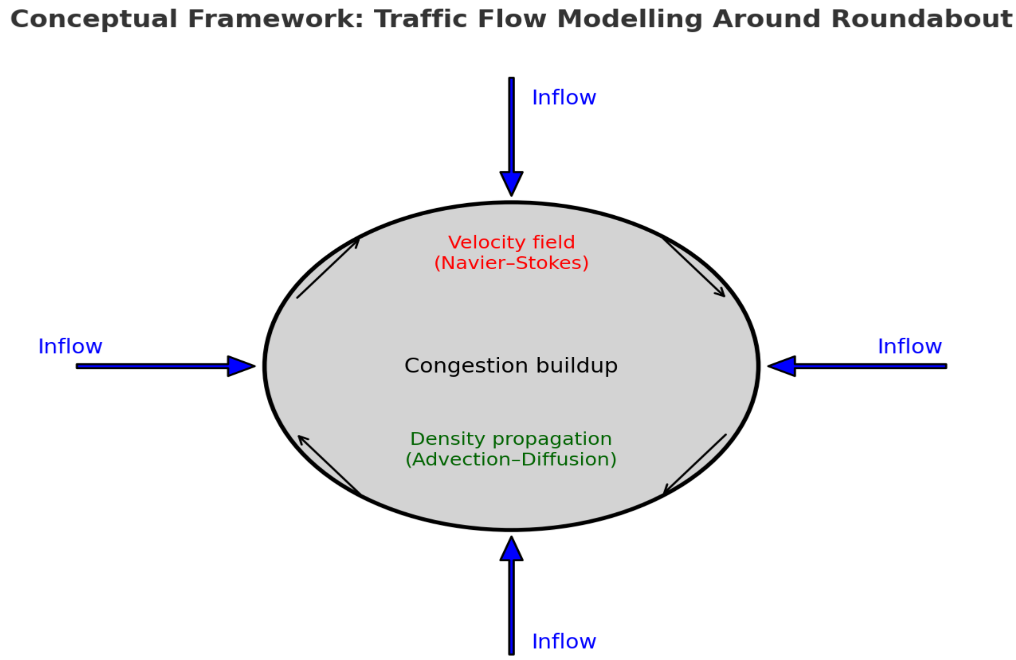

Fig. 2: Conceptual framework of traffic flow modelling around roundabouts using Navier–Stokes and Advection–Diffusion equations.

B. Empirical Review

Wang and Ruskin [4] addressed the challenge of accurately modelling traffic flow at single-lane urban roundabouts where traditional entry-capacity approaches were inadequate. They developed a multi-state cellular automata model under the offside-priority rule to simulate vehicle interactions and driver behaviors. The model categorized drivers into four behavioural types and analyzed throughput, queue lengths, and density changes. Findings revealed that throughput was influenced by roundabout topology, turning rates, and driver aggressiveness, but only marginally by geometric size. While the work advanced the microscopic modelling of roundabouts, it was constrained by its single-lane focus. The gap lies in extending these insights to multi-lane and continuum-based models using fluid dynamics, which motivates the proposed Navier–Stokes framework as a way forward.

Wang and Ruskin [5] extended earlier work by developing a Multi-stream Minimum Acceptable Space (MMAS) cellular automata model to simulate cross-traffic dynamics in unsignalized multi-lane roundabouts. The issue addressed was the inability of gap-acceptance models to scale beyond individual entries. Using heterogeneous driver categories and probabilistic reassignment, the model captured throughput and congestion characteristics under varying conditions. Results demonstrated that driver heterogeneity strongly affected flow stability and roundabout capacity. However, the critique is

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that the discrete approach lacked the analytical rigor of continuum models and struggled with parameter calibration. The research gap therefore lies in adopting Navier–Stokes and advection–diffusion formulations to generalize velocity and density dynamics across multi-lane facilities.

The work of Hoong and Hoe [7] investigated congestion in Malaysian roundabouts by employing macroscopic modelling through hyperbolic conservation laws. Their problem was rising vehicle numbersaveraging 519,000 new registrations annuallythat intensified delays and environmental costs. They applied the Godunov scheme to simulate three- and four-arm roundabouts, validating results against existing models and optimizing travel and waiting times. Findings indicated that their model yielded realistic traffic patterns and flexibility in handling different configurations. However, the study primarily remained macroscopic and country-specific, with limited focus on velocity field estimation or turbulence-like dynamics. This gap motivates the proposed adoption of Navier–Stokes equations, offering a more universal, physics-based approach.

Krogscheepers and Roebuck [8] explored unbalanced traffic volumes at roundabouts, noting that conventional models oversimplified by treating them as isolated T-intersections. Using TRACSIM simulations, they demonstrated how imbalances in approach flows severely altered delays depending on origin–destination patterns. Their results showed that capacity was highest under balanced flows but deteriorated sharply with imbalance. While significant in highlighting asymmetrical flow effects, the study lacked integration of fluid dynamic principles to predict velocity evolution under such conditions. The research gap is the absence of models that incorporate both spatial imbalance and continuum-based flow propagation. Addressing this limitation through Navier–Stokes-based modelling represents the way forward.

In recent work, Kabanga and Awuor [6] studied unsignalized circular roundabouts in Kenya using microscopic simulations grounded in the Intelligent Driver Model. The issue was persistent congestion at intersections in developing cities where traffic lights are absent due to cost constraints. Using MATLAB and Runge–Kutta numerical solutions, they simulated vehicle maneuvers and lane changes, identifying conditions that precipitate capacity breakdown. Findings highlighted the role of yielding failures and peak-hour surges in triggering congestion. However, the critique lies in the limited scale of simulations and absence of a fluid dynamic continuum perspective. The gap thus motivates the present study to apply Navier–Stokes and advection–diffusion formulations for velocity-focused modelling that can generalize across urban contexts.

C. Critique and Research Gap

TABLE I: SUMMARY OF RELATED LITERATURE ON TRAFFIC FLOW MODELLING AROUND ROUNDABOUT

Author	**Problem**	**Findings**	**Critique**	**Gap**
[4]	Inefficiency of single-lane entry-capacity models	Throughput depends on topology, turning rates, driver behavior	Limited to single-lane; ignores continuum effects	Need multi-lane, fluid-based modelling of velocity
[5]	Gap-acceptance models fail for multi-lane roundabouts	MMAS model captured heterogeneity and throughput	Discrete model lacks analytical rigor	Apply Navier–Stokes to multi-lane velocity dynamics
[7]	Malaysian congestion from rising vehicle numbers	Godunov scheme modelled 3–4 arm roundabouts	Macroscopic only; lacks velocity field focus	Introduce Navier–Stokes for velocity propagation
[8]	Unbalanced flows at roundabouts cause delays	Delays worsen under flow imbalance	Neglects fluid dynamics in velocity analysis	Integrate imbalance effects in Navier–Stokes framework
[6]	Congestion at unsignalized Kenyan roundabouts	IDM simulation showed yielding failures worsen congestion	Small-scale, lacks continuum modelling	Develop fluid-based velocity model for unsignalized flows

Table I show that existing studies persistent challenges in modelling traffic flow at roundabouts. Existing models capture driver heterogeneity, congestion, and imbalance effects but remain limited to single-lane, discrete, or macroscopic approaches. A research gap exists in fluid-based, Navier–Stokes modelling to analyze multi-lane velocity dynamics and unsignalized roundabout congestion comprehensively.

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III. PROPOSED METHODOLOGY

This study develops a numerical framework to model traffic dynamics at roundabouts using a coupled Navier–Stokes and Advection–Diffusion (NS–AD) formulation. The methodology integrates governing equations, discretisation strategies, non-dimensional analysis, and boundary/initial conditions into a systematic solver. The purpose is to capture critical behaviours such as vehicle velocity distributions, density propagation, congestion formation, and the impact of obstacles such as stalled vehicles inside the circulatory lanes.

A. Governing Model

The traffic field is represented by two coupled partial differential equations: the advection–diffusion equation for density ρ and the incompressible Navier–Stokes–like equation for velocity $\mathbf{v}$:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \mathbf{v}\cdot\nabla\rho &= D\nabla^2\rho + S, \\ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} &= -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{v} + k_{\text{obs}}\,\frac{\mathbf{r}}{\|\mathbf{r}\|}. \end{aligned} \tag{6}$$

The first equation governs vehicle density, where the left-hand side captures temporal change and convective transport, while the right-hand side includes diffusion (modelled by coefficient D) to represent random driver behaviour and the source/sink term S for entries and exits. The second equation governs the vehicle velocity field, where nonlinear convection reflects acceleration effects, the pressure gradient mimics density-induced resistance, and the viscous term $\nu\nabla^2\mathbf{v}$ provides numerical and physical stabilization. The obstacle force term models stalled vehicles or incidents, which are essential for understanding resilience of roundabout operations. Together, these equations form the backbone of the simulation framework.

B. Spatial Discretisation: Finite Volume Method with QUICK Scheme

The governing equations are solved on a uniform two-dimensional finite-volume mesh covering the roundabout and approach/exit links. The finite volume method (FVM) ensures local conservation of vehicle density and momentum by integrating the equations over discrete control volumes. Convective fluxes are evaluated using the Quadratic Upwind Interpolation for Convective Kinematics (QUICK) scheme:

$$\phi_{i+\frac{1}{2}} \leftarrow \frac{3}{8}\phi_{i+1} + \frac{3}{4}\phi_i - \frac{1}{8}\phi_{i-1}, \quad \phi \in \{\rho, \mathbf{v}\}. \tag{7}$$

This third-order interpolation reduces artificial diffusion and preserves sharp traffic density and velocity gradients, which are common in stop-and-go traffic conditions at roundabouts.

C. Temporal Integration: Crank–Nicolson Scheme

To advance the solution in time, a semi-implicit Crank–Nicolson (CN) scheme is employed:

$$\frac{\Phi^{n+1}-\Phi^n}{\Delta t} + \frac{1}{2}(\mathcal{C}^{n+1} + \mathcal{C}^n) = \frac{1}{2}(\mathcal{D}^{n+1} + \mathcal{D}^n) + \frac{1}{2}(\mathcal{S}^{n+1} + \mathcal{S}^n), \tag{8}$$

where $\Phi \in \{\rho, \mathbf{v}\}$, $\mathcal{C}$ denotes convection and $\mathcal{D}$ diffusion operators. The scheme balances efficiency and accuracy by treating diffusion and sources implicitly while maintaining consistency for nonlinear convection via QUICK. This improves numerical stability over explicit schemes and ensures second-order accuracy in time, which is critical for long simulation horizons.

D. Non-Dimensionalisation

Key parameters are extracted by scaling variables with characteristic density ρ_0, velocity U, and length L. This yields the Reynolds number, $\text{Re} = UL/\nu$, representing the relative importance of convective to viscous effects, and the source number, $\text{St} = U/(S_0 L)$, reflecting the balance between inflows/outflows and advection. Non-dimensional analysis provides a framework for interpreting flow regimes (e.g., advection-dominated versus diffusion-dominated) and guides the selection of stable mesh and time-step sizes.

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E. Boundary and Initial Conditions

Boundary conditions are defined to mimic realistic traffic entry and exit patterns. At inflow boundaries, density ρ and velocity $\mathbf{v}$ are prescribed based on approach demand, while at exits stress-free or specified discharge conditions are applied. No-penetration conditions are enforced along the splitter islands and central island, with tangential velocity guidance imposed to model circulatory movement. Initial conditions assume an empty roundabout with $v(x, y, 0) = 0$ and a low uniform density ρ, gradually ramped via S at entries to avoid numerical shocks. These conditions ensure physical realism and numerical stability.

F. Algorithmic Implementation

The numerical solver follows an iterative procedure:

1. Construct convective fluxes for density and velocity using QUICK interpolation.
2. Update density ρ with Crank–Nicolson, solving diffusion implicitly and advection semi-implicitly.
3. Advance velocity $\mathbf{v}$ using Crank–Nicolson with implicit viscous terms and QUICK-based nonlinear convection, while including obstacle forcing.
4. Update the pressure gradient based on the new density field, reflecting traffic compressibility effects.
5. Apply boundary conditions, check stability criteria (CFL numbers), and proceed to the next time step.

The simulation produces spatio-temporal fields $\rho(x, y, t)$ and $\mathbf{v}(x, y, t)$, from which performance metrics such as throughput, queue length, delay, and density wave propagation are derived. These outputs provide critical insights for evaluating roundabout design, operational resilience, and the effects of incidents or unbalanced inflows.

IV. RESULTS AND DISCUSSION

A. Parameter Estimation and Fitting

TableII lists the baseline numerical and modelling parameters used for all experiments. The set is intentionally simple and sufficient to reproduce the roundabout phenomena presented in (6) while keeping stability guarantees transparent and comprises a diffusion coefficient D, time step Δt, uniform grid spacings $\Delta x = \Delta y$, the simulated horizon t_{final}, and a lumped source term S that represents entries/exits or localized disturbances.

Table II: Optimal Values of Parameters used

Parameter	Description	Value Range	Value used	Source
D	Diffusion coefficient	-	0.1	[18]
dt	Time step size	-	0.01	Assumed
dx	Spatial x-step size > 1/2	-	1	Assumed
dy	Spatial y-step size > 1/2	-	1	Assumed
t_{final}	Simulation time	-	1	Assumed
S	Source term for disruptions	-	0.05	Assumed

B. Numerical Simulation and Discussion

We integrate the coupled NS–AD system on an annular roundabout domain with a conservative incident/body force $k_{\text{obs}}\mathbf{r}/\| \mathbf{r} \|$ and barotropic closure $p = a\,\rho$ ((6)). Convective terms in both ρ and $\mathbf{v}$ are discretised with a QUICK/TVD flux (MUSCL reconstruction with van–Leer limiter), so the scheme is high–order in smooth regions and reverts to monotone upwind near shocks. Diffusion and sources are advanced semi–implicitly with a Crank–Nicolson (CN) step; viscosity in the momentum equation is treated likewise. This pairing (QUICK + TVD + CN) delivers low numerical diffusion at the ring front while preserving stability over long horizons. Fig. 3 compares late–time density snapshots with and without an incident. The incident induces a persistent, bright annular ridge aligned with the circulating lanes; the baseline remains weakly modulated. The companion speed fields (Fig. 4) reveal a co–located amplification of $|\mathbf{v}|$ on the ring.

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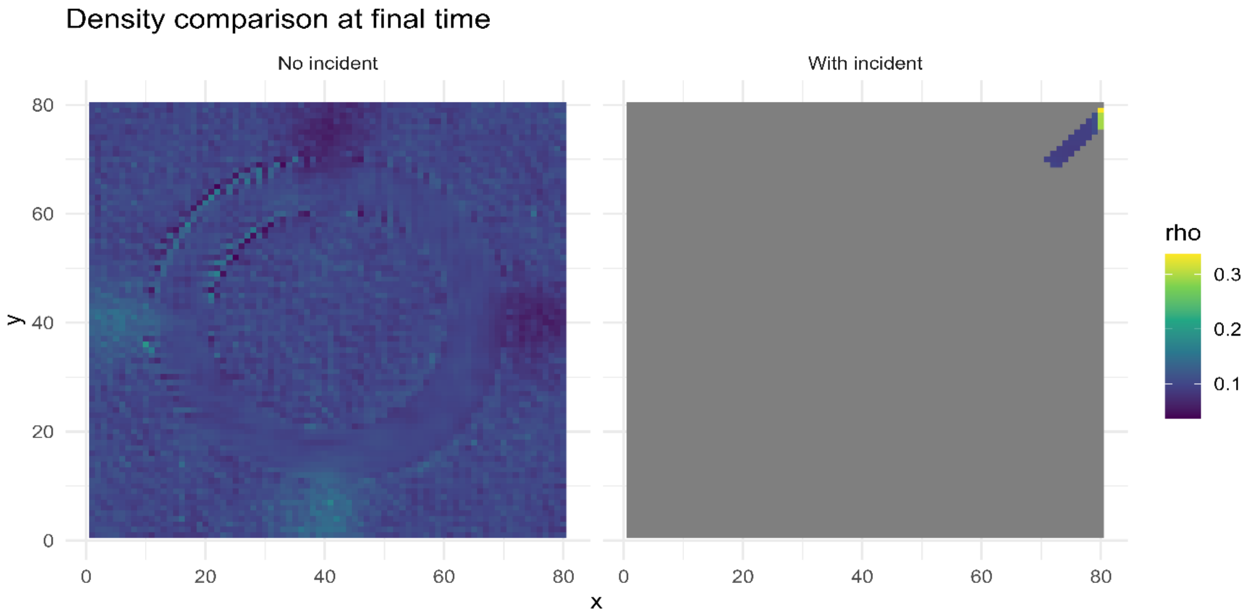

Fig. 3: Late–time density $\rho(x, y)$: *no incident* (left) vs. *with incident* (right). The incident produces a persistent annular congestion ridge.

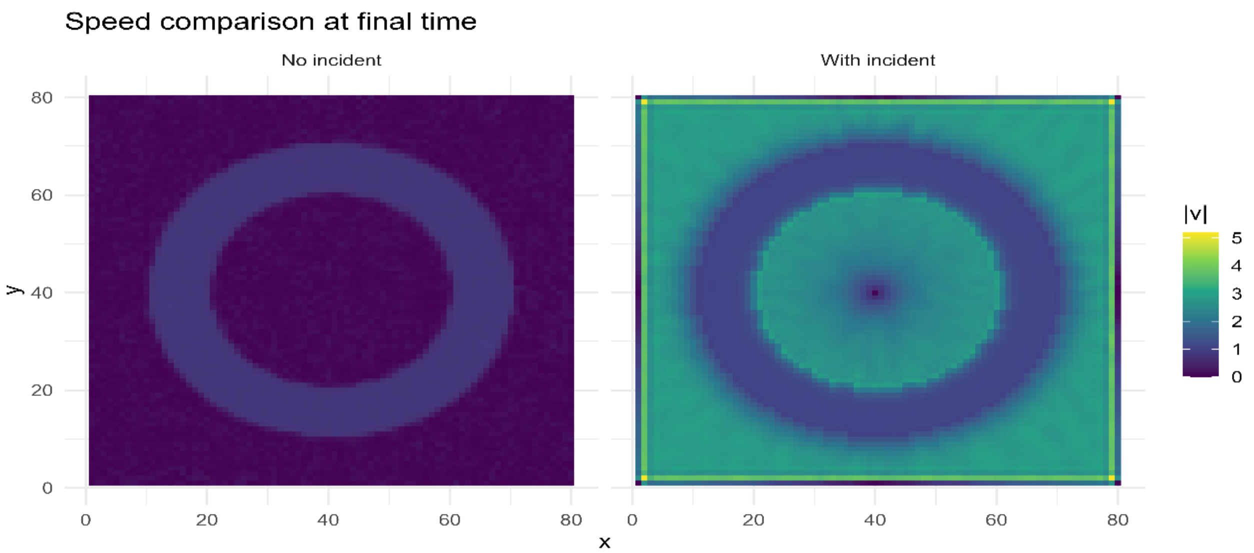

Fig. 4: Late–time speed $|\mathbf{v}|(x, y)$ for the two scenarios. The incident strengthens speeds along the ring and imprints the upstream/downstream braking/acceleration asymmetry.

Because $p = a\rho$, $-\rho^{-1}\nabla p = -a\,\nabla(\ln\rho)$. Along the ring, the sign of the tangential gradient sets the local acceleration: $\partial_\theta \rho > 0$ (approaching a denser patch) brakes the flow, whereas $\partial_\theta \rho < 0$ (leaving a denser patch) accelerates it. Hence the observed sandwich of low/high–speed streaks that flank dense sectors in the incident runs and the enhanced magnitudes in Fig. 4. At probe B (outer band), $\rho_B(t)$ declines in both scenarios, but the incident produces a markedly steeper, near–linear increase in $|\mathbf{v}|_B(t)$ (Fig. 5–6). The joint evolution traces a monotone phase curve with negative slope (Fig. 7), i.e. as the local density relaxes, speed increases. A parsimonious explanation is a quasi–steady balance of inertia against the composite potential $a\ln\rho + \Phi$ with $\nabla\Phi = k_{\text{obs}}\mathbf{r}/\|\,\mathbf{r}\,\|$: as ρ falls, the pressure drift weakens and larger speeds become admissible under essentially unchanged forcing.

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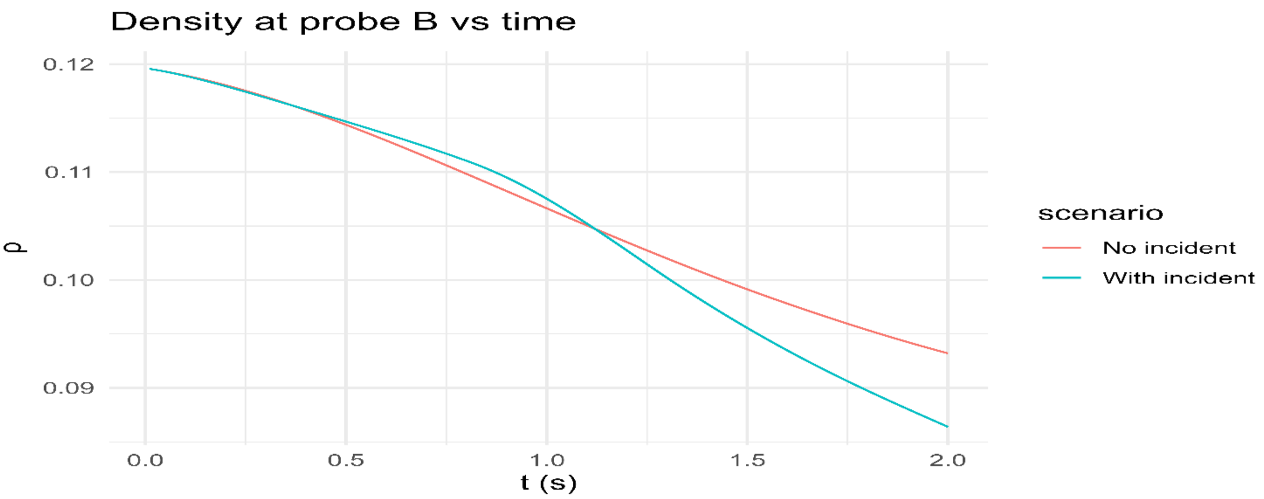

Fig. 5: Probe–B density $\rho_B(t)$. Both cases evacuate; once the ring organises, the incident re–routes flow and hastens local depletion.

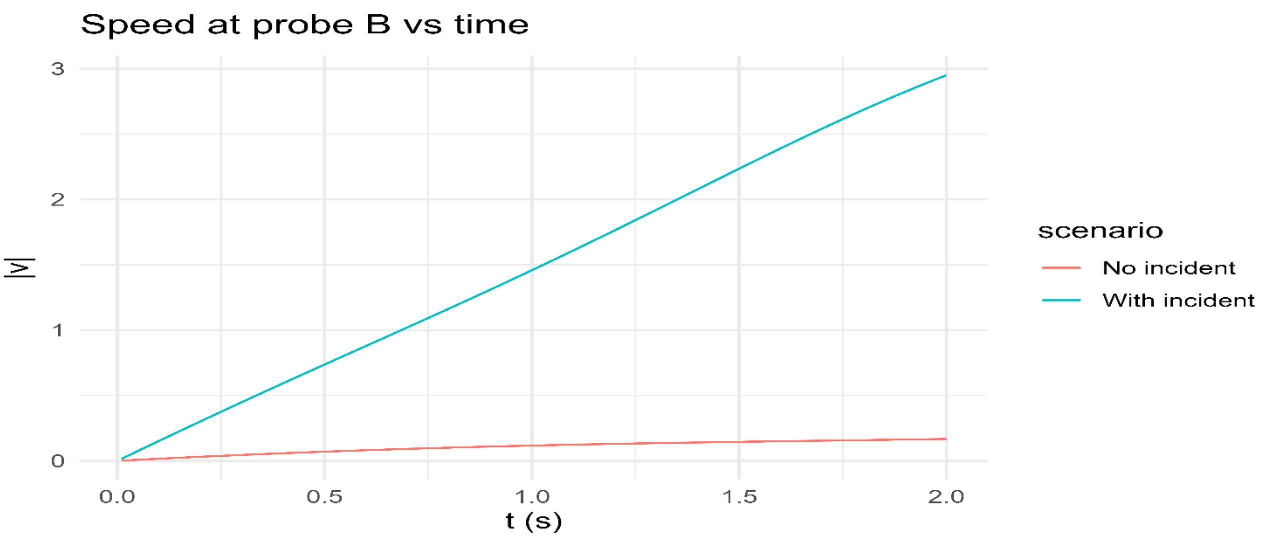

Fig. 6: Probe–B speed $|\mathrm{v}|_B(t)$. The incident produces a stronger, near–linear speed–up throughout the simulated horizon.

On a mean radius R the dominant ring dynamics reduce to

$$\partial_t \rho + \frac{u}{R}\,\partial_\theta \rho = \frac{D}{R^2}\,\partial_\theta^2 \rho + S(\theta), \tag{9}$$

$$\partial_t u + \frac{u}{R}\,\partial_\theta u = -\frac{a}{\rho R}\,\partial_\theta \rho + \frac{\nu}{R^2}\,\partial_\theta^2 u. \tag{10}$$

Balancing advection and diffusion across a steady front gives the angular thickness $\delta \sim D/u$, predicting sharper wedges for larger u and broader wedges for larger D, which matches the behaviour seen in our parameter sweeps. With U a typical ring speed and L a band width, the Péclet and Reynolds numbers, $\mathrm{Pe} = UL/D$ and $\mathrm{Re} = UL/\nu$, place the density equation in an advection–dominated regime and the momentum equation in a convection–viscous balance. The QUICK+TVD flux is thus essential to avoid nonphysical oscillations at the sharp annular front while maintaining resolution of the wedge; CN furnishes second–order accuracy in time and stability for long runs. The incident consistently (i) creates a pressure–supported annular congestion ridge, and (ii) strengthens speeds on that ring. This suggests: rapid clearance of incidents near the central island; entry metering on feeder approaches that directly load the ring; and geometric/control measures (short bypasses, advisory speeds) that reduce the effective barrier $a\ln\rho$ and provide additional diffusion pathways, thereby lowering ridge amplitude and smoothing the transient.

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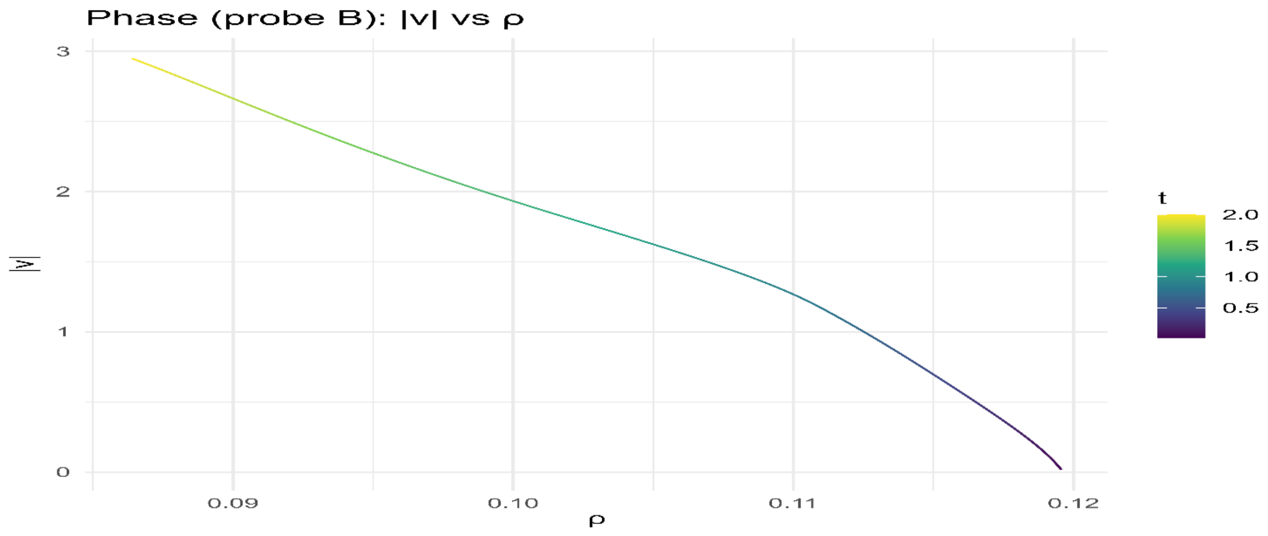

Fig. 7: Phase portrait at the probe: |v| versus ρ (coloured by time). The negative slope evidences a local anti–correlation as the transient unfolds.

C. Discussion

Our simulations integrate the NS–AD system on an annular roundabout with a conservative incident/body force $k_{\text{obs}}\mathbf{r}/\|\mathbf{r}\|$ and barotropic closure $p = a\,\rho$ ((6)), advanced with QUICK/TVD (MUSCL +van Leer) for convection and Crank–Nicolson for diffusion/viscosity. Two robust signatures emerge: (i) a persistent annular *congestion ridge* in ρ for the incident case (Fig. 3), and (ii) a co–located amplification of $|\mathbf{v}|$ with a characteristic upstream/downstream braking/acceleration asymmetry (Fig. 4). At a ring probe, density decays while speed rises nearly linearly, producing a negative $(|\mathbf{v}|, \rho)$ phase slope. These patterns are natural consequences of the closure $-\rho^{-1}\nabla p = -a\nabla \ln\rho$ and the conservative incident potential, and they provide a continuum, physics-based lens for results reported across microscopic and macroscopic empirical studies.

Wang and Ruskin [4] showed, in a multi-state cellular automata (CA) model for single-lane urban roundabouts, that throughput depends sensitively on topology, turning rates, and driver aggressiveness. In our framework, the same drivers of variability enter as *source geometry* and *azimuthal heterogeneity* in $S(\theta)$ and as *effective stress* parameters (a, ν) that encode how velocity responds to density and how momentum diffuses around the ring. The simulated annular ridge is a continuum analogue of the dense platoons seen in microscopic models; its angular thickness obeys $\delta \sim D/u$, so sharper (broader) wedges align with greater (smaller) advective strength, mirroring how aggressive driving or high turning demand narrows (or widens) high-density sectors in CA studies.

Extending to MMAS cellular automata, Wang and Ruskin [5] found that driver heterogeneity strongly affects flow stability and capacity under unsignalised multi-lane operations. Our NS–AD formulation generalises this by letting the *same* PDE govern the entire ring while heterogeneity appears as spatially varying coefficients and sources: lane-to-lane interactions can be represented by multi-species densities $\{\rho^{\ell}\}$ sharing momentum through cross-diffusion/drag terms, or in the single-field case, by structured $S(\theta)$ that injects or removes mass at specific approaches. The simulated speed amplification downstream of dense sectors (set by $\partial_\theta \rho < 0$) offers a mechanistic explanation of the local "stability pockets'' reported in MMAS: low pressure drift $-a\nabla \ln\rho$ permits higher speeds where density relaxes.

Hoong and Hoe [7] used Godunov discretisations of hyperbolic conservation laws to reproduce observed delays for three- and four-arm roundabouts, concluding that macroscopic models capture realistic patterns and are flexible across layouts. Our results are consonant: the *same* ring-level balance

$$\partial_t \rho + \frac{u}{R}\partial_\theta \rho \;=\; \frac{D}{R^2}\partial_\theta^2 \rho + S(\theta), \qquad \partial_t u + \frac{u}{R}\partial_\theta u \;=\; -\frac{a}{\rho R}\partial_\theta \rho + \frac{\nu}{R^2}\partial_\theta^2 u$$

governs the annular dynamics. Where the Godunov approach advects ρ with a fundamental diagram, NS–AD additionally resolves $|\mathbf{v}|$ and its diffusion, enabling spatio–temporal velocity estimation and ridge-thickness prediction ($\delta \sim D/u$) that are difficult to extract from scalar laws alone.

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Krogscheepers and Roebuck [8] showed that approach imbalances degrade capacity and inflate delay. In our simulations, imbalances map directly to nonuniform $S(\theta)$ and induce sustained azimuthal $\partial_\theta \rho \neq 0$, which (via $-a\,\partial_\theta \ln\rho$) creates alternating braking/acceleration streaks and relocates the annular ridge toward over-supplied sectors. Thus, the empirical sensitivity to OD patterns corresponds to a precise PDE mechanism: an azimuthal pressure gradient that redistributes momentum and mass.

Kabanga and Awuor [6] identify yielding failures and peak-hour surges as triggers of capacity breakdown in Kenyan roundabouts. In our model, such breakdowns correspond to transient increases in S and/or to localised incident forcing k_{obs}, which steepen $\nabla\ln\rho$ and produce the observed negative $(|\mathbf{v}|, \rho)$ slope at probes. The predicted downstream speed amplification and thin high-throughput corridors (right after dense sectors) are testable signatures in probe or video data and suggest targeted speed harmonisation just *before* dense sectors.

Across the empirical record, key phenomena, dense platoons, capacity loss under imbalance, and recovery/downstream acceleration, are reproduced by the NS–AD solver with additional structure: (i) a composite potential $a\ln\rho + \Phi$ explaining outward migration and metastable ring formation; (ii) quantitative ridge width $\delta \sim D/u$; (iii) explicit velocity fields $\mathbf{v}$ and their diffusion (ν); and (iv) a principled way to encode incidents through Φ.

V. CONCLUSION

Roundabouts alleviate conflict points and improve safety, but they remain vulnerable to recurrent congestion when approach demands are unbalanced or when incidents occur within the circulatory lanes. Traditional modelling, either microscopic with detailed driver rules or macroscopic with density-only dynamics, has struggled to deliver a single, predictive apparatus that jointly resolves velocity and density, represents incidents mechanistically and provides calibration-ready quantities. By formulating traffic on the roundabout as a coupled NS–AD system with barotropic closure $p = a\rho$ and a conservative incident force $k_{\text{obs}}\mathbf{r}/\|\mathbf{r}\|$, and by advancing it numerically with QUICK/TVD convection and Crank–Nicolson diffusion/viscosity, we recover the principal empirical signatures of roundabout congestion: a pressure-supported annular ridge in density aligned with the circulating ring and a co-located speed amplification that exhibits upstream braking and downstream acceleration dictated by the sign of the azimuthal density gradient. Probe time series reveal a negative local $(|v|, \rho)$ correlation during transients, while a quasi-1D azimuthal reduction explains the organisation of the ridge and produces the scaling $\delta \sim D/u$, linking wedge thickness to diffusion and ring speed. Direct field differencing between incident and baseline runs isolates the causal effect of the incident as outward mass migration and momentum gain along preferred escape paths, confirming that a composite potential $a\ln\rho + \Phi$ governs outward push, pressure support and viscous/diffusive smoothing. These findings support several concrete conclusions: the continuum framework is expressive enough to capture the kinematic imprint of incidents and imbalance; the numerical pairing achieves resolution without oscillations over long horizons; and the resulting diagnostics are suitable for empirical calibration and operational decision-making. From a policy perspective, rapid clearance of near-island incidents, targeted metering at approaches that directly load dense sectors and the deployment of short bypasses or advisory speeds emerge as cost-effective levers because they reduce the effective barrier $a\ln\rho$ and create additional diffusion pathways that temper ridge amplitude and moderate speed surges. Looking forward, the model should be calibrated to site-specific data to infer $(a, D, \nu, S(\theta), k_{\text{obs}})$, extended to multi-lane and multi-class contexts with cross-diffusion and lane-change source terms, equipped with real-time data assimilation and model-predictive control for metering and harmonisation, and coupled to emissions and safety surrogates so that operational strategies can be evaluated on mobility, environmental and risk metrics within one coherent, physics-based framework.

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